Plato's Ghost

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These are some notes on [Gra08]. This is a book that argues that around the end of the first world war mathematics underwent a significant and profound *cultural* change, that parallels the similar occurances in literature and fine arts called "modernism".

Its title "Plato's ghost" is an allusion to a poem by Yeats in which Platos ghost appears to always remind the artist striving for perfection that they have not yet accomplished it. It is also an allusion to the philosophy of Platonism that Gray sees as an outcome of the modernist turn.

What I want from this book is the following: I too feel after reading through the history of mathematics that what we are doing now is culturally and psychologically very different. It is also clear that this turn began around the begining of the 20th century. I think that there was also a big change after the second world war as well with Bourbaki, Grothendiek and Leray, but first things first is understanding how the mathematical psychy was changing in the first half of the century. At this stage the primary literature is huge and I cant sift through it all myself, sans taking a lot of time off, thus this book is hopefully something like an answer to these questions but having someone else go through the literature for me. Indeed my project reading through the history was to understand questions of foundations and ontology as seen in the past and this book is advertised as giving essentially an account of this aspect of mathematics in the early 20th century.

Introduction

He surveys some definitions of modernism. He lists the following features

> . Freedom from the constraint of juries, critics or any law making art-body, involving 2. The rejection of most of the traditional ideas of art, even to the denial that beauty is worth seeking. As this seems opposed to the principle of evolution, and is only negative, I do not see how it can be maintained. 3. Interest in the expression of each individual, whether it conforms to a school or not, whether it be agreeable or the reverse. 4. Less attention to subject, more to form. Line, mass and color have pure aesthetic value whether they represent anything or not. Ceasing to make representation a standard but comparing the visual arts with music. Finding a common basis for all the visual arts. 5. Convincing us that there are limitless fields yet unrevealed by art. C. Lewis Hind says that "Matisse flashes upon canvas the unexplored three-fourths of life." 6. New expression by color, not by the colors of things, or color in historic art. Seeking hitherto unexpressed relations of color. 7. Approaching, through non-applied design and in other methods the creation of new types of design, decoration and craft work.

He argues that it was the formation of a socio-economic class of "mathematician" that is the formation of the professional, that lead to this transformation. That reflection and attempts at differenting this class from that of "physicist" or "philosopher" are core causes. That it "emerged through the first coherent group of researchers, as opposed to gifted individuals".

He signposts that he will argue for a fundamental change in mathematical ontology and in turn changes in epistemology. namely the subject matter becoming "more autonomous" lead to changing standards of proof.

He tells us that he will essentially argue that the different modernist transformations were independent, although the different people involved may have been aware of one another the fundamental causes were not related.

He then spends some time discussing the previous literature on this, in particular Mahrtens.

1 Chapter 1

This is still a somewhat introductory chapter. The beginnings of the argument is fleshed out a little more however.

Historically the goals of mathematics have not been solely the rigorization of the field. Other goals and motives are correct reasoning (as its own goal) proving theorems, resolving contradictions, good applications and pedagogy. At the turn of the century however rigour was most prominent.

The introduction of non-Euclidean geometry, in my opinion only for historical reasons, ended up deeply undermining the relationship between mathematics, science and philosophy. This was compounded by "the crisis of continuity" which was the discovery that there were continuous functions that were highly non-intuitive, i.e. non-physical.

In addressing the problem of analysis and continuity Gray argues that there were two complimentary approaches. First the ontological approach where the real numbers were attempted to be grounded in the rational numbers, and in turn they in the natural numbers, this is the arithmetisation of analysis. This in turn leads to questions about formalising the natural numbers themselves, which is answered by set theory. The other epistemological approach was needed due to the issues of continuity, these strange functions could not be physical and so began the separation between analysis and mechanics (as a realm of knowledge). These issues again would be addressed by set theory.

While in analysis the idea of the integers was being reduced to sets etc, there was another direction in which the concept of number was being assailed. Namely that of algebra. In these times (1800's) a number system was something like the integers or the reals or the complex numbers. Hamiltons introduction of the quarternions, which proved to be fruitful and as valid as the introduction as the complex numbers, produced something that was *like* the integers but in all the wrong ways. Thus the somewhat naive way of thinking of number systems was in need of revision.

> Ontologically the erosion of the concept of number is crucial.

These issues were addressed Gray argues in a modernist fashion. In geometry by Hilberts axiomatics, free of intuitive geometric definitions, in analysis by set theory and in algebra by abstract structural definitions.

Another development at this time was the interest in mathematical logic, or logic per se. Gray argues that the interest was two fold. One was to develop it further, as Euclidean geometry was not written in syllogisms we needed an extension and clarification of just what logic was. The other was the hope that whatever the resulting answer was, would suit as a new foundation for the mathematics that was beginning to be torn from its traditional roots in physics. Thus logicism was born. Unfortunately as is well known the end result of this story is "the modernist transformation of logic itself" which mean that "even logic had no straightforward connection to simple clear thinking".

1.1 Philosophy Before

Why was the non-Euclidean geometry such a silver bullet to the philosophy of mathematics at the time. The answer is the dominance of Kantianism. The dominant philosophy at the time was a series of "nuanced responses and rejections of Kant" of course this always comes with swallowing a great deal of Kant. Kants philosophy relied on the fact that space was *necessarily* Euclidean, with the acceptance of non-Euclidean geometries this statement no longer feels *necessary*. Thus geometry was no longer about space as it is and therefore was no longer about anything.

One of the major responses to Kant was to go back before him, to Liebniz, this leads to logicism and formalism. To consider mathematics only as a language, this was a view in the eighteenth century (Liebniz, Lambert, Condorcet) that was also revived.

> Two intertwined themes governed thinking about mathematics in the period. One concerned its truth, the other (to be discussed below) the nature of mathematical reasoning. Put simply, the prevailing view around 1800 among mathematicians was that mathematics was true... The validity of these truth claims was contested by the discovery and gradual acceptance of non-Euclidean geometry. The nature of its objects was thereafter found to be less and less clear, until the traditional hold on them, was lost.

The end result of this debate however was "the philosophy of mathematics was siply left in disarray", as "every original protagonist" was taken out by historical contingency. Thus around the 1920's and 1930's although logicism was to some extent found to be untenable, no conclusive answer was given to what is tenable. Gray argues that out of this, Platonism was the dominant philosophy that emerged somewhat organically exclusively in the 20th century.

1.2 Professionalisation

Gray argues, on historical grounds, that before 1900 mathematicians concerned with foundations more or less wrote directly for one another, that is published or letters that were intended to be read only by a very few select peers. At the turn of the century Hilbert, Poincare and Enriques popularised the subject by bringing it into the *mathematical* zeit geist. This was made possible by the general trend that saw mathematics as a whole go from a small community that wrote only for a few other people to a large community with large popular publishing outlets and schools, particularly in Germany.

The shift took place over the 19th century, and further divided the mathematicians from the physicists as they searched for an independent identity to justify their independent positions at universities and ignorance of experiment. As the professoriate grew individuals also seek an identity with in, and Gray argues that this created a motivation for research for its own sake, which pushed mathematics to be more abstruse, and forced mathematicians to keep on the cutting edge.

> Mathematics was placed in a complicated rather than a naive relationship with the day-to-day world. It was not derived from the world in any simple way, and it was not necessarily applied to it in any simple way.

2 Chapter 2

This chapter is a historical survey of analysis, algebra, geometry and logic through the period 1800 − 1880. It is mostly uninteresting for my purposes. Ill note a few things.

Around 1820's projective geometry was invented and popularised. This could be seen as a precursor or parallel development to non-Euclidean geometry because it was in some sense more general than Euclidean geometry. He argues that Euclidean geometry was entrenched by the education system, history and Kantian philosophy. Its acceptance was aided later on when a new generation of students came along and were less tied up with the past views.

Another thing to note is that the French revolution reshaped French universities into something more modern.

There was a sense in the 1800's that arithmetic was god given, geometry was closer to mechanics than mathematics and analysis was the somewhat dominant form of mathematics that had its own process of formalisation.

Another point he recognises is that the development of algebraic number theory was also challenging the concept of number. Kronecker Kummer etc were trying to extend the idea of prime to numbers in what we would call now algebraic integers. This was controversial at the time because to make the definition of prime work in these new number system Kummer was postulating the existence of number that were not in $\mathbb{C}$ effectively.This was his "ideal numbers" which of course evolved into modern ideals of rings.

The discussion of logic is somewhat enlightening so I will put a little more detail here. He emphasises that syllogistic logic was calcified as THE mode of reasoning. Boole and De Morgan in Britan started a bit of a modern resurgence in logic (along with others). Statements with quantifiers were realised to not only be not syllogistic but to be ubiquitous in mathematics. For instance

> all equilateral triangles are equiangular triangles.

Boole sums up at least how I used to feel about logic, that "all elimination could be effected by syllogism but only after the original statements had been re-expressed using non-syllogistic methods". It was Peirce in America, not Frege, that made the first steps to introducing quantifiers in logic.

3 Chapter 3

At this point I want to recall a few points I didnt record earlier as well as recapitulate what has come before. This chapter is the history of the really early stages of the modernist turn.

3.1 Context

The first thing that I want to cover is the general historical context in which what can be called modern mathematics as a whole (before modernism) grew up, that is the period after 1800. The first American presidential election was held in 1789, the Loisianna purchase, which double the then American countrys land mass, restricted as it was to the east coast, was in 1804. The first French revolution was in 1789, the Napolionic wars against Britain, Germany, Russia, Hapsburgs (and everyone else) were waged through 1803 - 1815. France underwent several other revolutions, flipping between monarch and repulblic and variously waging wars with mainly Germany. This was a very different, much smaller world. There was very little connection to scholars of the east.

In this context Gray estimates during the period 1830 - 1860 that there was approximately a total of four to five hundred total mathematicians in the western world (2.1.4) (for comparison there is this discussion on estimates of today, depending on your definition etc but one reasonable order of magnitude would be 100,000).

One reason to point this out is that mathematics was much more susceptible to a few things. One is the influence of individuals, it is easier to convince 500 people of something than 100,000. Put another way if you could convince 10 of your friends of something then you had captured two percent of all mathematicians. Another reason that this is important is that these few people were much more localised and culturally homogeneous. By population they were dominated by the French and German, with sizeable but comparably small populations in Britain, Russia and other European states. A knock on effect of these things is that mathematics is much more susceptible to the changes of the period. Only a few people had to become engaged in whatever popular philosophy for it to potentially have massive effects, or if Germany felt a cultural shift it would find its way into the culture of mathematics. For instance Gray argues

> Germany's experience at the start of the nineteenth century, culminating as it did in defeat by Napoleon, had convinced them that applied-led research was too narrow. The intellectual response was the philosophy of neohumanism, which argued that doing pure mathematics for its own sake was not only best for that subject but also best for those who would want to apply it.

Now this population was growing and its constituents becoming more diverse from this period onwards, however the general fact is still true. By 1900 Germany and France still dominated and the total population would still be very small in the hundreds or low thousands.

3.2 Summary

Before 1880 (roughly) modernism had appeared in a few places, mainly associated with Riemann. The dominant philosophy was some psudeo-Kantian philosophy. Discoveries in the dominant fields of mathematics, logic (Boole and Pierce working on logic as a foundation and the inadequacy of syllogism), analysis (Weirstrass and many others arithmetising the subject and finding "strange" functions), algebra (Kummers work on Fermats last theorem and attempting to introduce new integers) and geometry (Riemann and Pasch with non-Euclidean and projective geometry) were wll forcing the previous paradigms to be questioned. This opened the way for a lot of original and "free" thought. Once the very foundations are questioned all becomes open for contemplation and thus we see many fundamentally different views of mathematics beginning to emerge.

3.3 1880 - 1900

With this list of issues from the beginning of the century in mind the new generation of mathematicians enter the scene. The new players were Klein, Kronecker, Cantor, Dedekind, Clifford, Killing, Heine in Germany Poincare in France and Peano in the rest of the world.

Not only was there adoption of non-Euclidean geometry there was some repudiation of Euclidean geometry. "The compliment paid to Euclid's Elements by mathematicians of all cultures, which consisted of putting right the imperfections that had turned up while regarding the whole structure as fundamentally sound, was ceasing to be paid." He provides a scathing quote from Pasch. In respect to the unsound foundations of the time a view typical of this period is that of Enriques

> As to those intuitive concepts, we do not intend to introduce anything other than their logical relations, so that a geometry thus founded can still be given an infinite number of interpretations, where an arbitrary meaning is ascribed to the elements called "points." [However,] we think that the experimental origin of geometry should not be forgotten while researching those very hypotheses on which it is founded.

That is geometry needed to be exercised from the Euclidean intuition in all its formal aspects, however those intuitive concepts should remain as one of the potential applications of the newly created mathematics. This same view was taken by Paul Du Bois-Reymond who had the motto ”empiricist language, idealist proofs”. Reymond was using the distinction that was ontological, the idealist was able to admit the actual infinite, limits etc, whilst an empiricist would only admit that things in nature really exist. Benno Kerry argued a very modernist (maybe formalist?) point of view that neither of Reymonds alternatives were necissary, existence was merely the absence of contradiction (this is somewhat Liebnizian). Gray writes about Kerry's ideas

> It did not seem to Kerry that the existence of a limit in either the idealist or the empiricist senses that du Bois-Reymond had invoked mattered in mathematics, unless one wanted it to be true and applicable.
>
> Du Bois-Reymond, Kerry noted, had an exclusively geometrical sense of existence in mind, but the way in which geometric points exist should not be confused with the way in which limits exist. Things exist in different ways, and failure to exist in one sense does not preclude it in another.
>
> Du Bois-Reymond's book bears witness to a deeply held belief that mathematics must be about something, its objects should exist, and should do so in a way closely akin to the way physical objects do. Existence should mean something like existence in space and time. Kerry's alternative was much more radical. Existence is freedom from contradiction, mathematical objects may exist in many ways and have merely to imply coherent conclusions.

This debate about existence was weighed in by Kronecker who was apparently a proto-constructivist "It is not enough say "Either a thing is, or it is not". One must show what one wants to be and what not to be in the particular domain with which we are concerned". Non-contradiction is not enough a construction is necessary.

Poincare would make many contributions. One thing that he and Klein both advocated was geometries connection to group theory (the connection is the different geometries are essentially captured by the groups that preserve the relevant structure), Gray argues that "Poincare always felt that groups came before spaces because our knowledge of space was derived from our knowledge of the behaviours of rigid bodies". Meanwhile in analysis Poincare said that "matter does not engage their attention, they are interested by form alone", when talking about the foundations of the real numbers.

In this period Dedekind, Cantor, Heine and Meray all came up with essentially the same definition of real number, as sets of rationals. Both Dedekind and Cantor were absolutely key to the furtherance of set theory. First Dedekind developed the beginnings of finite set theory in ”What are numbers and what are they for?" moreover he also had used the same concept to make Kummers ideal numbers rigorous. Dedekind had defined ideal numbers as *collections of numbers*, these were ideals in the modern sense. Note that Kronecker had an alternative formulation of ideal numbers, and Gray posits that it is largely due to the support of Hilbert that Dedekinds theory came down to us. Cantor on the other hand had discovered properly infinite sets and in his theory of cardinal arithmetic had found a new system of numbers. This was highly controversial, but the key point is that it led to an investigation of the foundation of numbers and arithmetic, as all the usual operations straightforwardly extend to transfinite ordinals. Gray puts it as follows

> It is therefore amusing, and typical of the modernist shift, that just as two theories of real numbers were proposed that removed doubt from the foundations of the calculus differences of opinion opened up about the integers.
>
> The highly contentious claim to have discovered transfinite numbers required at the very least a clear account of finite numbers, so that the new ones could legitimately be seen as sufficiently numberlike to merit their name.

Cantors proof of the existence of a bijection between the unit interval and the unit square provided further issues for the foundations of geometry as it called into question the notion of dimension.

Hegelianism was in decline and philosophers such as Trendelenburg had contributed to the revival of Liebniz. This is how Frege was to learn of the Characteristic Universalis. He famously tried to define numbers and made contributions in logic (although not as great as I had once thought). Frege did not like the idea of infinite sets and instead opted for "concepts", this lead to Russells paradox in his indelicate handling of them. Concepts make it more clear that mathematics and logic are a part of only rational thought, thus allowing mathematics to be subsumed by logic. On the other hand Dedekind (and Cantor to an extent) had a parallel theory in which *sets* formed the foundation, they were the "immediate products of pure thought" and thus could in fact form the basis of a logicist program.

Remark. Cantor is much painted as an outcast, in fact many people, including Poincare, were supporters of his early work on the reals, sets and Fourier transforms. What was controversial was his work on transfinite cardinals and his claim that they had the same status as numbers.

Remark. Apparently there was a contemporary modernist turn taking place in the catholic church as well that ironically wholly embraced Cantor and his work on the real infinite.

4 Chapter 4

This chapter is dealing roughly with the period 1900 - 1910. One thing that surprised me is how many of the mathematicians I thought were long dead were still alive, Cantor and Frege spring to mind. Not only alive but active. The other thing is how highly connected the community was at this time. I had a sort of idea that there was no internet so no one knew each other. No, there were lots of letters flying back and forth, articles in journals responding to other peoples articles and conferences were all the people that we have mentioned met.

Hilbert is a famous formalist, with his famous quote being that "points lines and planes should be replaceable by tables chairs and beer mugs". We have heard this sentiment in the last chapter, that many people thought that mathematics should be somehow grounded in geometric language but that that language should be used only according to general laws such that it was valid for any substitution. The idea is that geometric objects were no longer clearly defined and so the language should be about anything. Hausdorff was an enthusiastic supporter, wanting mathematics to be free of all intuition.

Poincares philosophy became called conventionalism. The question that Poincare was dealing with was how could one decide between Euclidean and non-Euclidean geometry. Poincare argued that their logical validity was essentially equivalent, that any contradiction derived in one could be transported to the other, thus one could not decide between the two on logical grounds. In the realm of physics he similarly argued that they were indistinguishable. The main point of contention in space is what is a straight line, you either define it as the path light takes in which case space is curved or you define space as flat in which case the path of light is curved, there is no way to distinguish. Thus one merely had to make an arbitrary choice and stick with it, just pick a convention as it were.

The flag of intuition was taken up by Poincare, Borel and Klein, as opposed to formalism. They felt that in research it would always be necissary to be guided by intuition. Klein said

> I do not grant that the arithmetized science is the essence of mathematics . . . it is not possible to treat mathematics exhaustively by the method of logical deduction alone, but that, even at the present time, intuition has its special province

He agreed that mathematics could to an extent, and maybe even should, be formalised but that this process would never capture the essence of mathematics. There philosophy was more holist

> They point quite clearly toward a problem that has not gone away in philosophers' treatments of mathematics: a tendency to reduce it to some essence that not only deprives it of

> purpose but is false to mathematical practice. The logicist enterprise, even if it had succeeded, would only have been an account of part of mathematics—its deductive skeleton, one might say. Living mathematics, as it is actually done, would remain to be discussed.

This position had an appeal to the Kantians who were revived, and argued that of course space and time were intuitions.

Analysis in the hands of Borel, Baire, Frechet, Riesz, Lebesgue and others became heavily set theoretic, with the invention of measure theory. It was also taking on an axiomatic character, say with the notion of integral being freed from its intuitive definition in terms of area. This can be seen as a supplanting of the concept of area by another "measure" and not necessarily an anti realist etc position however. This work along with that of Hausdorff was foundational to the emerging discipline of topology.

Gray then concludes the chapter with a long discussion of the paradoxes of set theory. One funny quote is that appon the discovery "Bernstein later recalled that Dedekind began to doubt if human thought is completely rational". It is interesting that many people knew about it before Frege and Russell, in particular Zermelo wrote on it at least two years before. However "For Zermelo it was merely one of those difficulties at the frontier that would eventually have to be sorted out." It was not a crisis, as it was for Frege and Russell who had based their logicist philosophy on it. Zermelo in 1904 published a paper proving that every set can be well ordered, this precipitated a great debate over what operations or what things we could reasonably do with sets. It was in response to this that Zermelo came up with an axiomatisation of set theory so that this proof could be more naked in its assumptions.

> All of these questions echoed a broader problem which had rarely been enunciated explicitly: What methods were permissible in mathematics? Must such methods be constructive? If so, what constituted a construction? What did it mean to say that a mathematical object existed? Normally mathematicians avoided such quasi-philosophical questions, and addressed them only when they felt their discipline to face a crisis. That Zermelo's proof precipitated such a crisis was shown by the extent of the resulting controversy.

These axioms largely were adopted because of their usefulness and the fact that people simply lost interest, not in the axiomatic methods, but in the details of whether this or that operation was truly logically necessary for the concept of a set etc. "The debate did not end, it was not resolved".

Remark. Just some unimportant but interesting historical remarks. Hilbert restructured number theory to rely heavily on Galois theory. Peanos main contribution was his notation, mixing logic and set theory, his school used this notation for arithmetic and geometry. Husserl was actually a big deal philosophically around this time. Poincare never said anything that disparaging about set theory, only commenting that it was at the time riddled with contradiction, he even proposed his solution of predictive definitions. The main influence of Principia Mathematica was again as Peano simply the notation that they had invented for their logical calculus.

5 Chapter 7

Chapter 5 and 6 describe mostly philosophical positions that are not interesting to me or this study right now. Im sure that the history contained there, including a history of the history of mathmatics, is interesting and fruitful to study but I dont really care about the relation of maths and physics at this stage as I think that it is essentially clear that the two were by this point highly distinct. Chapter 7 on the other hand is the final historical part talking about the interwar period 1920 - 1940.

Some general remarks about the effect of the (first world) war. Apparently 40% of young French mathematicians were either killed or wounded. On the other hand Britan and Germany did not use its intellectuals on the front line and so suffered far fewer casualties. Thus Germany still thrived intellectually after the war, despite its economic trouble. This dynamic would switch in the lead up to the second world war, as France recovered and the Nazis would dismantle the intellectual capacity of Germany. Before the war the philosophy of mathematics and foundations were experiencing unprecedented popular attention, after the war the general public had other problems. Thus the intellectual life receded back to a much more speciality field again.

5.1 Proof Theory

The field of proof theory was more or less instigated by Hilbert. It obviously underwent a massive expansion in this period with Godel, Turing, Church, Post, Skolem etc all active at this time. There was also a younger generation than Hilbert including Godel, Turing, Heyting, Von Neumann, Weyl etc. that would take these issues into the future.

For the purposes of philosophy the main debate was between Hilbert and Brouwer. Afterwards thought of as a "tempest in a tea cup" the debate was in some sense quite fruitful. In the end Hilbert would use his position at a journal to force Brouwer off the editorial committee after which Brouwer would retire from academia in disgust.

The content of the debate is well trodden ground, however we will make a few remarks. Gray correctly frames this as a debate ultimately about finitism and the limits of thought, not merely a disagreement about the law of excluded middle. At this point Hilberts formalist philsophy could sort of be phrased as follows, the things that he considered primitive were "extra-logical discrete objects which exist intuitively as immediate experience before all thought", the idea that these things were the marks on the page. All else should follow from logic applied to these primitive marks formed from almost *pure intuition*.

Now the main issue at this time was what rules were valid to apply to infinite sets. Hilbert's idea is that we could replace the infinite sets with the finite reasoning about them. Thus the intuition about finite sets and finite marks became a priori knowledge for Hilbert. Brouwer on the other hand was essentially a mystic. What can be made concrete from his thinking is that he essentially just rejected that any abstract reasoning about the infinite could be done, and instead only reasoning about the concrete and finite could be considered valid. This is the constructive side of thought, if you wanted to show something about say an infinite set, that it contained an element say, then you needed to take a single thing and perform some reasoning to show that it belonged to the set. Thus reasoning about the infinite set was replaced by reasoning about one thing. Gray puts the two men in their place as follows

> Hilbert was a free spirit , optimistic and saw no limitation on the human mind when it came to mathematics. Brouwer saw only limitations on the human mind.

5.2 Set Theory

Zermelos first attempt at axiomatisation as we saw was in 1908. His axiomatisation did not allow for instance the construction of the ordinal $\aleph_\omega$. Fraenkel was part of the new generation of mathematicians and wrote some papers on Zermelos axioms trying to remedy this fact. He introduced an axiom of replacement in 1922, Von Neumann showed in 1928 that actually Fraenkels new axiom followed from Zermelos original. After this Zermelo once again began to work on the axioms and it was Zermelo who ironically fixed Fraenkels axiom of replacement. He thus suggested a new axiomatisation in 1929 and 1930. These are apparently more or less what we have now, although I have not checked that.

5.3 Platonism

Gray has unfortunately little to say about the actual development of this stance. He makes a few apt comments however.

> mathematicians deal with philosophical questions about mathematics uneasily, and this is further manifest in the ill-defined term "Platonism" with its vague, unanalyzed connotations of Plato's theory of forms.

> Another motive may also have been at work. The more mathematics was defined as conceptual mathematics, the more it lost contact with ordinary quantity, the more it lost its sense of reference. Restoring objects in mathematics may have offered reassurance to mathematicians who felt themselves cut off from the classical subject they had studied in their youth.

Indeed the point about Platonism in modern mathematics can mostly be put as follows, mathematicians dont have a philosophy, they have a vague feeling that they are talking about something however and since the 19th century what exactly that is is very unclear, so they just say "Something" with a capital now. This philosophy did have serious defenders notably in Godel, however Godels Platonism is very hard to articulate for me and I would say certainly not relevant to an analysis of the contemporary zeit geist. Thus I guess the suggestion is that Platonism is somehow the most intellectually lazy philosophy that one can grab a hold of, not to say that one cannot defend it in an honest way but that one doesnt need to. Moreover it provides ready and naive answers to the questions that mathematicians might have and moreover requires no changing of their habits as say Brouwers intuitionism would have. Platonism is the path of least resistance.

6 Summary

Im realising I never spell check any of the notes I make. Anyway what did we learn from this book. What summary of mathematical philosophy can we now make in the period 1880 - 1920. Unfortunately the thesis of Grays work is that mathematics became modern and in essence became *complex*. The naive philosophies, epistemological and ontological that were once tenable were no longer tenable. The responses were nuanced, idosyncratic and manifold. Heres my attempt at a synthesis.

Mathematics in this period and just before can, for statistical reasons, be conflated with mathematics in France and Germany. The dominant philosophy was that of Kant, who regards space and time as a priori intuitions, he regarded geometry as synthetic a priori, plainly a necessary truth. With the work of Riemann, Gauss, Pasch and others the necessity of geometry as it was seen by Kant, that is Euclidean geometry, was no longer obvious. Thus mathematics was cut loose from the philosophical tradition and all hell broke loose. Note that Kants philosophy provided a metaphysics and epistemology.

At the same time an epistemological anxiety developed in the 19th century. This was independent of the non-Euclidean revolution but certainly compounded with it. The anxiety was precipitated by a collapse of mathematicians faith in their faculty of naive intuitive reasoning. This was due to the many unintuitive results proved in this period, think of things like Cantors proof that there is a bijection between the real line and the plane (thus dimension is no longer clear), space filling curves, continuous but no where differentiable curves, logical puzzles from Boole and Peirce around how mathematics could or could not be written in syllogism, the usefulness of ideal numbers in algebra etc. All of this made the idea that mathematicians were capable of naively going about their work and deriving true statements dubitable.

As a resolution to both of these problems we see the resurgence of the axiomatic method. People wanted to formalise definitions to no longer rely on intuitive words or concepts but to basically be pinned down by a list of properties that the thing must satisfy. From the axioms and logic then one could derive truths about anything that satisfied those axioms. The axiomatic method however was cause for pause, what were axioms about, or when were axioms about something. This was a major question that basically was answered in every conceivable way. For instance one can be a realist like Frege, that the axioms are true properties of real things, the things that perhaps we would previously have defined in terms of concepts. One could say that the axioms were mere placeholders, they did not assert the existence of anything, but could be applied to anything that happened to exist and satisfied them because deductions made from them would have been valid. Others simply said that the axioms described what they described and as long as the axioms were consistent then whatever that was existed, maybe not in a physical sense but in a mathematical sense.

This axiomatic approach is to me the most lasting effect of this period. The philosophies all lived and died, the axioms themselves rose and fell, but this method, this insistence that we must take a collection of ground truths and develop everything else up from them has remained. In different fields different things are taken for granted, say in representation theory we assume vector space exist and certain facts about them etc, in topology there might be a different grab bag of basic facts that everyone agrees on. Usually these things are given point set justifications but at base the working mathematician is axiomatic in their approach, not naively applying intuition, but applying it to some collection of assumed truths about axioms.

The axiomatic method generated more questions and debates around set theory geometry and algebra didn't slow down for decades. Things like what was the role of intuition in deriving things from them, what was the role of the infinite, what was the logic that one should be allowed to apply to axioms, are these things all related. Again these were all answered in a myriad of ways. The result of this however was merely fatigue, compounded by wars, ending in ultimately forgetting. People did take whatever tools that came out of them, say proof theory, the notation of Principia etc, and treated them naively like anything else and created a new field. The philosophy was just left in the dirt, unresolved and from it grew the ready answer of essentially naive Platonism. The questions now have somewhat conventional answers but they are not deeply rooted. Naivety reigns.

7 Further Reading

- Guillaume Appollinaire - The beginings of Cubism
- Charles Sanders Pierce
- Volkert - The crisis of intuition
- Moritz Eple (1999)
- Pasch - Projective geometry
- Corry - 1996, 2007
- Helmholtz, Kronecker (philosophy)
- d'Alembert - "traditional view of philosophy of mathematics"
- Benacerraf 1973
- Michell 1993, 1999
- Boole - Mathematical analysis of logic, Laws of thought
- Killing - On the FOundations of Geometry
- Thomae - an early formalist
- Borel 1898 - First textbook on set theory
- Van Der Waerden - Modern algebra 1930 - Corry 1996 for discussion of the development of algebra.
- Hausdorff - Grundzuge der mengenlehre 1914

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